

## CHANNEL ESTIMATION TECHNIQUES FOR MIMO-OFDM AND SISO-OFDM SYSTEMS

**DIVYA SHARMA, SHRIPAD P. MOHANI & PARAG JOSHI**

Department of Electronics and Telecommunication, College of Engineering, Pune, Maharashtra, India

### ABSTRACT

In this paper, the block-type pilot channel estimation for orthogonal frequency division multiplexing (OFDM) and Multiple Input Multiple Output (MIMO) OFDM systems is investigated. The estimation is based on the minimum mean square error (MMSE) estimator and the least square (LS) estimator. The MMSE and LS estimators are elaborated and their performances are compared. Our results show that the performance of MMSE estimator is better but its complexity is high, contrary to the LS estimator that has low complexity but poor performance. We evaluate estimator's performance on the basis of mean square error and bit error rate.

**KEYWORDS:** Block Type, Channel Estimation, Least Square (LS), MIMO-OFDM, Minimum Mean Square Error (MMSE)

### INTRODUCTION

Orthogonal Division Multiplexing (OFDM) is based on the principle of dividing a high bit rate data stream into many parallel lower data rate streams that are transmitted simultaneously over a number of subcarriers, which are orthogonal to each other [5], thereby increasing the symbol duration and reducing the inter symbol interference (ISI). The Multiple Input Multiple Output (MIMO) technology employs multiple transmit and receive antennas at either end of the wireless link to increase data rates or the reliability with which data is received [9]. MIMO-OFDM technology is therefore promising way to deliver the high data throughput and quality of service projected for future wireless systems. MIMO systems increase the data rate without expanding the bandwidth, increase the diversity and improve the performance against fading channels using space-time codes [10]. MIMO-OFDM systems requires channel characteristics in order to correctly demodulate the received data by employing coherent detection schemes so reliable channel estimation is needed. The most important challenge of MIMO-OFDM systems is how to obtain the channel state information accurately for coherent detection of information symbols [8]. The channel state information can be obtained through different types of estimation algorithms such as training based (Pilot based), blind and semi-blind channel estimation.

The blind channel estimation methods require a large amount of data and the convergence rate is very low. Hence, it goes against the real-time channel estimation [4]. The training based channel estimation can be performed by sending training symbol or Pilot symbol either by Block type or Combo type pilots. In Block type pilot estimation, pilot's tones are inserted into all frequency subcarriers within the periodic intervals of OFDM blocks and it is employed under the assumption of slow fading channels only. But in Combo type pilot estimation, pilot tones are inserted into each OFDM symbol within the specific periods of frequency bins. In this paper, channel estimation based on Block type pilot estimation is carried out.

## SYSTEM DESCRIPTION

The SISO-OFDM system is shown in figure 1, in which the incoming binary bit streams are first modulated using QAM modulation techniques resulting in modulated signal  $\mathbf{X}(k)$ . The modulated signal  $\mathbf{X}(k)$  is employed with IFFT to obtain time domain signal. IFFT is used in OFDM system to achieve the orthogonal property between the subcarriers. Here,  $N$  subcarriers transform into  $N$  point IFFT. The time domain representation of IFFT signal is  $x(n) = \text{IFFT}[\mathbf{X}(K)]$ .

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) \cdot e^{j2\pi k \frac{n}{N}} \quad (1)$$

Cyclic prefix is a replica of the last data samples of OFDM signal that is inserted at the start of the OFDM symbol. It is added to IFFT signal  $x(n)$  to preserve orthogonality and secure against multipath effects of channel for a time period of useful data in signal. The signal that is resulted after adding guard interval is given as

$$x^g(n) = x(n + n_g) \text{ where } n_g = -N_g, -N_g + 1, \dots, -1 \text{ \& } n = 0, 1, 2, \dots, N-1 \quad (2)$$

Where  $N_g$  is the length of the guard interval. The transmitted signal will pass through the frequency selective time varying fading channel with additive noise. Then, the received signal is

$$y^g(n) = \{x^g(n) \otimes h(n)\} + w(n) \quad (3)$$

Where channel impulse response can be expressed as

$$h(n) = \frac{1}{\sqrt{N}} \sum_m \alpha_m e^{-j\frac{\pi}{N}(k+(N-1)\tau_m)} \frac{\sin \pi \tau_m}{\sin(\frac{\pi}{N}(\tau_m - k))} \quad (4)$$

Where  $\alpha_m$  is the  $m^{\text{th}}$  complex path gain,  $\tau_m$  is the corresponding path delay, and  $T_s$  is the sampling interval. At the receiver end, guard interval is removed from the received signal.

$$y^g(n + n_g) \text{ For } -N_g \leq n \leq N - 1 \quad (5)$$

The signal becomes  $y(n)$ , which is then applied to FFT block to transform the signal into frequency domain, which gives  $Y(k)$ , where

$$Y(k) = \text{FFT}\{y(n)\} \text{ k}=0, 1, 2, \dots, N-1 \quad (6)$$

Then, the signal at receiver is expressed as

$$Y(k) = X(k) H(k) + W(k) \quad (7)$$

Where,  $W(k)$  is zero mean noise  $W(k) = \text{FFT}\{w(n)\}$  and  $H(k)$  represents channel frequency response, where  $H(k) = \text{FFT}\{h(n)\}$ .

MIMO-OFDM system is shown in figure 2. Here, MIMO-OFDM system with two transmit antennas ( $N_t$ ) and two receive antennas ( $N_r$ ) is considered. The MIMO-OFDM system is fed with input data which is digitally encoded to increase the security of transmission, minimize errors at the receiver, or maximize the rate at which data is sent. The input bit sequence is converted into a sequence of complex symbols using QAM modulation. The modulated signals are then space-time coded using the Alamouti algorithm [6] before transmitting from multiple antennas [7], not necessarily implemented jointly over all the  $N_t$  branches. The MIMO-OFDM transmitter has  $N_t$  parallel transmission paths, which are very similar to the single antenna OFDM system, each branch performing serial-to-parallel conversion, pilot insertion,

N-point IFFT and cyclic extension before the signal is finally transmitted. At the receiver side, the CP is removed and N-FFT is performed per receive branch. After this, the transmitted symbol per transmitted antenna is combined for digital demodulation and decoded in order to recover the transmitted binary bits.

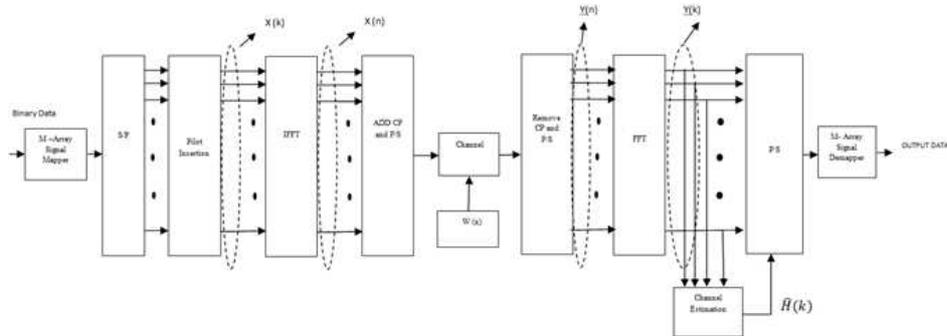


Figure 1: SISO OFDM System Block Diagram

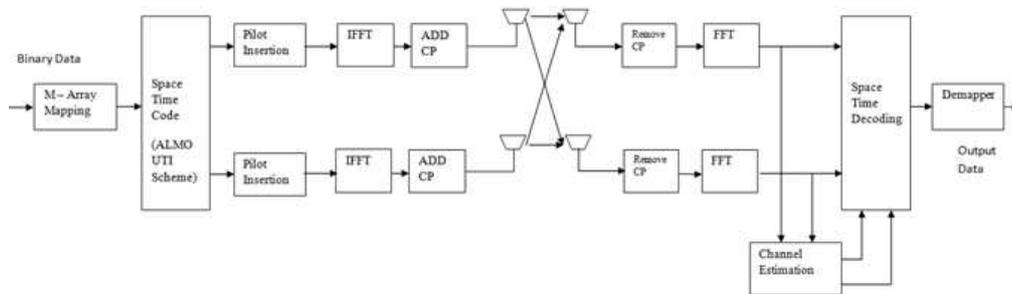


Figure 2: MIMO OFDM System Block Diagram

In MIMO system,  $N_t$  different signals are transmitted simultaneously over  $N_t \times N_r$  transmission paths and each of those  $N_r$  received signals is a combined effect of all the  $N_t$  transmitted signals and the noise. The system capacity is increased by the diversity, but when compared to the SISO system, it complicates the system design regarding channel estimation and symbol detection due to the hugely increased number of channel coefficients. The data stream from each antenna undergoes OFDM Modulation. The Alamouti Space Time Block Coding (STBC) scheme has full transmit diversity gain and low complexity decoder, with the encoding matrix represented as referred in [11].

$$X = \begin{bmatrix} X1 & -X2 \\ X2 & X1^* \end{bmatrix}$$

$$X1 = (X [0] -X^*[1] X[2] -X^*[3] \dots \dots -X^*[N-1] )$$

$$X2 = (X [1] X^*[0] X[3] X^*[2] \dots \dots X^*[N-2] ) \tag{8}$$

The vectors  $X1$  and  $X2$  are modulated using the inverse fast Fourier transform (IFFT) and after adding a cyclic prefix as a guard time interval, two modulated blocks  $X_{g1}$  and  $X_{g2}$  are generated and then transmitted by the first and second transmit antennas respectively, with an assumption that the guard time interval is more than the expected largest delay spread of a multipath channel. The received signal will be the convolution of the channel impulse response and the transmitted signal. Assuming that the channel is static during the time span of an OFDM block, at the receiver side after removing the cyclic prefix, the FFT output as the demodulated received signal can be expressed as

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_{N_R} \end{bmatrix} = \begin{bmatrix} H_{1,1} & H_{1,2} & \cdots & H_{1,N_T} \\ H_{2,1} & H_{2,2} & \cdots & H_{2,N_T} \\ \vdots & \vdots & \ddots & \vdots \\ H_{N_R,1} & H_{N_R,2} & \cdots & H_{N_R,N_T} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_{N_T} \end{bmatrix} + \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_{N_T} \end{bmatrix} \quad (9)$$

In the above equation  $[W_1, W_2, \dots, W_{N_T}]$  denotes AWGN and  $H_{m,n}$  is the (single-input single-output) channel gain between the  $m^{\text{th}}$  receive and  $n^{\text{th}}$  transmit antenna pair. The  $n^{\text{th}}$  column of  $\mathbf{H}$  is often referred to as the spatial signature of the  $n^{\text{th}}$  transmit antenna across the receive antenna array. By knowing the channel information at the receiver, Maximum Likelihood (ML) detection can be used for decoding the received signals for two antenna transmission system, which can be written as

$$\begin{aligned} \tilde{s}[k] &= \sum_{i=1}^{N_r} H_{i,1}^* [2k] Y_i [2k] + H_{i,2} [2k] Y_{i,1}^* [2k + 1] \\ \tilde{s}[2k + 1] &= \sum_{i=1}^{N_r} H_{i,2}^* [2k + 1] Y_i [2k] - H_{i,1} [2k + 1] Y_{i,1}^* [2k + 1] \end{aligned} \quad (10)$$

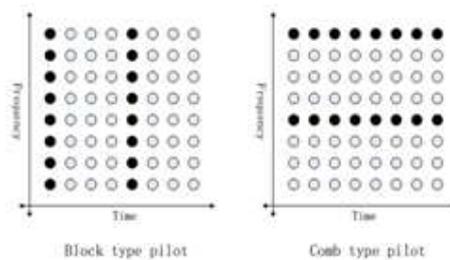
With an assumption that channel gain between two adjacent sub channels are approximately equal,

$$\begin{aligned} H_{i,1}[2k] &= H_{i,1}[2k + 1] \text{ \&} \\ H_{i,2}[2k] &= H_{i,2}[2k + 1] \end{aligned} \quad (11)$$

At the end, the elements of block  $\tilde{s}[k]$  are demodulated to extract the information data.

## CHANNEL ESTIMATION

The pilot channel estimation methods are based on the pilot channel and pilot symbol. Channel estimation is a technique in which pilot sequence and pilot symbols are inserted into some fixed positions of signals sent by transmitter [2]. The pilot symbol sent by the transmitter makes spectral efficiency and power utilization lower with the trade-off of quick response to the channel variation. For pilot based channel estimation of OFDM system, suitable pilot pattern needs to be considered, Pilot-based channel estimation algorithm with low complexity should be identified and proper demodulation method toward effective channel estimation has to be developed. The two basic channel estimations in OFDM systems, block-type pilot and comb-type pilot, are illustrated in Figure 3.



**Figure 3: Block and Comb Type Pilot**

The block-type pilot channel estimation is performed by inserting pilot tones into all subcarriers of OFDM symbols with a specific period in time and it is effective against the selective frequency fading, but more sensitive for the impact of fast fading channel [3]. Therefore, the block-type pilot is developed under the assumption of slow fading channel. In this paper, the block-type pilot channel estimation is investigated. The comb-type pilot channel estimation is performed by inserting pilot tones into certain subcarriers of each OFDM symbol, where the interpolation is needed to estimate the conditions of data subcarriers.

**SISO OFDM CHANNEL ESTIMATION**

For block-type pilot based channel estimation, we can estimate the channel using LS (Least square) Estimator and MMSE (Minimum mean square error) Estimator. In block type, we are inserting the pilot on each subcarrier after a specific period. The impulse response of multipath channel can be written as

$$h(t, \tau) = \sum_{k=0}^{L-1} \partial_k(t) \delta(t - \tau_k) \tag{12}$$

Where  $\tau_k$  the delay of  $k^{th}$  path is,  $\partial_k(t)$  is the amplitude of  $k^{th}$  path, and  $L$  is the number of sub-carriers, respectively. Because  $\partial_k(t)$  is a stationary narrowband complex Gaussian process, it is independent of each other for different paths. If there is no ISI, the signal received is expressed as

$$Y = XF\hat{h} + W \tag{13}$$

Where  $Y$  is the vector of output signal after OFDM demodulation as  $Y = [Y_0, Y_1, \dots, Y_{N-1}]^T$ ,  $X$  is the diagonal matrix of pilots as  $X = \text{diag}\{X_0, X_1, \dots, X_{N-1}\}$ ,  $N$  is the number of pilots in one OFDM symbol,  $\hat{h}$  is the impulse response of the pilots of one OFDM symbol, and  $W$  is the channel noise (always assumed to be AWGN. Also,  $F$  is the Fourier transform matrix as shown below:

$$F = \begin{bmatrix} W_N^{00} & \dots & W_N^{0(N-1)} \\ \vdots & \ddots & \vdots \\ W_N^{(N-1)0} & \dots & W_N^{(N-1)(N-1)} \end{bmatrix}$$

Where  $W_N^{i,k} = \frac{1}{\sqrt{N}} e^{-j2\pi(\frac{ik}{N})}$  (14)

**LS Estimator for SISO OFDM System**

The purpose of LS algorithm is to minimize the  $(Y - XF\hat{h})^H (Y - XF\hat{h})$  & generates the channel estimator as

$$\hat{H}_{LS} = X^{-1}Y \tag{15}$$

Where  $\hat{H}$ , is the impulse response of the channel  $\hat{H} = F\hat{h}$ .

The advantage of LS algorithm is its simplicity, because of no consideration of noise and ICI[1]. So, without any knowledge of the statistics of the channels, the LS estimators are calculated with very low complexity, but obviously, it suffers from a high mean square error (MSE).

**MMSE Estimator for SISO OFDM System**

The MMSE estimator employs the second-order statistics of the channel conditions to minimize the MSE. Let us denote the error of channel estimation  $e$  as

$$e = H - \hat{H}$$

Where  $H$  is actual channel estimation and  $\hat{H}$  is raw channel estimation, respectively. Let us denote the auto-covariance matrices of  $H$  and  $Y$ , by  $R_{HH}$  and  $R_{YY}$  respectively, and cross covariance matrix between  $H$  and  $Y$  by  $R_{HY}$ . Let  $\sigma_N^2$  is the noise-variance, since the channel and AWGN are not correlated, we could get

$$R_{HY} = E\{HY^H\}$$

$$R_{YY} = E\{YY^H\}$$

$$R_{YY} = XR_{HH}X^H + \sigma_N^2 \quad (16)$$

If  $R_{HH}$  and  $\sigma_N^2$  are known to the receiver, channel impulse response can be calculated by MMSE estimator as shown in equations 17 to 19.

$$\hat{H}_{MMSE} = R_{HY}R_{YY}^{-1}Y \quad (17)$$

$$\hat{H}_{MMSE} = R_{HH}X^H(XR_{HH}X^H + \sigma_N^2I_N)^{-1}X\hat{H}_{LS} \quad (18)$$

$$\hat{H}_{MMSE} = R_{HH}(R_{HH} + \sigma_N^2(X^HX)^{-1})^{-1}\hat{H}_{LS} \quad (19)$$

The performance of MMSE estimator is much better than LS estimator, especially under the lower  $E_b/N_0$ . And MMSE estimator could gain 10-15 dB more of performance than LS. However, because of the required matrix inversions, the computation is very complex when the number of subcarriers in OFDM system increases. Thus, it is seen that an important drawback of the MMSE estimator is high computational complexity.

## MIMO-OFDM CHANNEL ESTIMATION

### LS Estimator for MIMO-OFDM System

The LS channel estimation for MIMO-OFDM System shown in figure 2, between the  $n^{\text{th}}$  ( $n=1, \dots, N_t$ )

Transmitter antenna and  $m^{\text{th}}$  ( $m=1, \dots, N_r$ ) receiver antenna is given by:

$$\hat{H}_{LS}^{(n,m)} = (X^{(n)})^{-1}Y^{(m)} \quad (20)$$

### MMSE Estimator for MIMO-OFDM System

The MMSE channel estimator between the  $n^{\text{th}}$  antenna and  $m^{\text{th}}$  antenna is given by

$$\hat{H}_{MMSE}^{(n,m)} = R_{HH}^{(m,n)} [R_{HH}^{(m,n)} + \sigma_N^2((X^{(n)})(X^{(n)})^H)^{-1}]^{-1}\hat{H}_{LS}^{(n,m)} \quad (21)$$

## SIMULATION AND RESULTS

The simulation process enables the analysis of different channel estimator performance to find the optimal channel estimator with low complexity. The system parameters for OFDM simulation are indicated in the table given below:

**Table 1: Simulation Parameters**

Parameters	Specifications
IFFT/FFT size	64
No. of Subcarriers	64
Signal Constellation	16 QAM
Pilot ratio	1/8
Guard Length	8
Guard Type	Cyclic Prefix
Pilot	Block Type

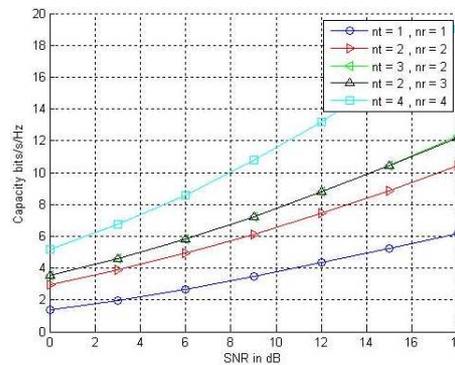
The programs are executed in MATLAB simulator and the model validations are done on the basis of analysis of two parameters that are MSE and Bit Error Rate (BER). The MSE of an estimator is one of many ways to quantify the

difference between the theoretical values of an estimator and the true value of the quantity being estimated. MSE measures the average of the square of the error. The error is the amount by which the estimator differs from the quantity to be estimated. We define the mean square error as

$$\text{Mean square error} = \text{mean} [\{ \text{abs} (H) - \text{abs} (\hat{H}) \} ^2]$$

Where,  $H$  is theoretical transfer function and  $\hat{H}$  is the calculated transfer function for each estimator.

BER is the number of bit changes made to the transmission medium per second using a digitally modulated signal. Figure 4 shows the Channel Capacity v/s SNR plot for various number of receive and transmit antennas. The impact of the antenna number on the channel capacity is examined. As expected, the capacity bounds increase monotonically as SNR increases. Channel capacity increases by 2 bps/Hz for every 2 dB SNR increase for MIMO and it increases by 1 bps/Hz for every 2 dB of SNR increase for SIMO, MISO or SISO (at high SNR).



**Figure 4: Channel Capacity v/s SNR for SISO and MIMO OFDM System**

The BER v/s SNR plot comparison for LS and MMSE Estimator for SISO-OFDM is observed in Figure 5. At a certain point around BER= 0.01, there is approximately 10 dB difference in SNR between LS and MMSE estimator. We can see here that performance curves of these two systems are close to each other when SNR is low, but gap gets larger when SNR gets higher. Similarly, for the plot of MSE v/s SNR for SISO-OFDM which is averaged over 1000 OFDM blocks at every SNR value as shown in Figure 6. It can be observed that there is performance improvement from system to system and also estimation algorithm wise. However, MMSE Estimator shows low values of MSE even for small SNR values, whereas LS Estimator has higher values of Mean Square Error.

Next, we have implemented MIMO-OFDM system and in our scenario, we are using a 2x2 MIMO system. Figure 7 shows us the comparison between LS and MMSE estimator on the basis of BER v/s SNR plot for MIMO OFDM system. We can observe from the plots that for smaller values of SNR (for 5 dB to 20 dB) for MMSE estimator, the BER values range from  $10^{-5}$  to  $10^{-6}$ , whereas for LS estimator, the BER values range approximately around  $10^{-3}$ . For higher values of SNR, the BER values decrease gradually for both the estimators, but BER values for MMSE estimator are smaller compared to those for LS Estimator.

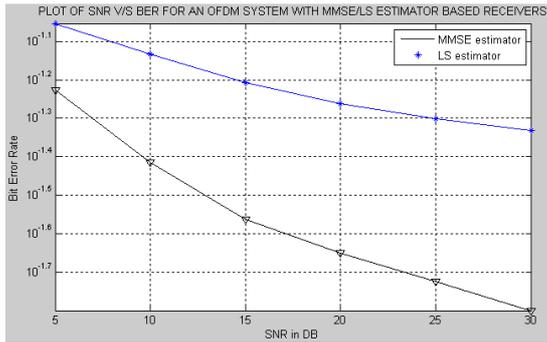


Figure 5: BER v/s SNR for SISO OFDM System

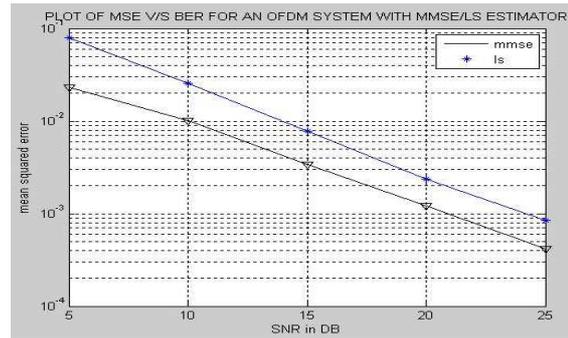


Figure 6: MSE versus SNR for SISO OFDM System

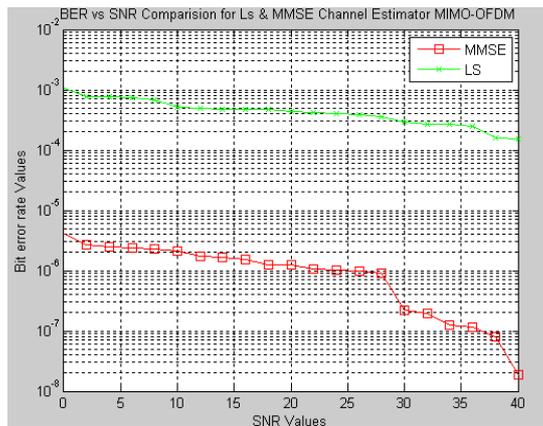


Figure 7: BER v/s SNR MIMO OFDM System

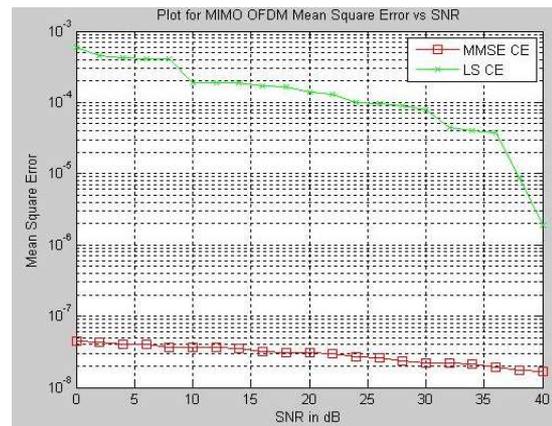


Figure 8: MSE versus SNR for MIMO OFDM System

Figure 8 shows the MSE v/s SNR plot for 2x2 MIMO-OFDM system. Here, the performance analysis for both the estimators LS and MMSE for SNR range from 0 dB to 40 dB is shown. LS estimator MSE is from  $10^{-3}$  to  $10^{-5}$  and for MMSE estimator, the range of MSE is about  $10^{-7}$ . It can be seen that MMSE estimator offers the lower MSE compared to LS estimator.

**CONCLUSIONS**

In this paper, block type pilot based channel estimator for SISO-OFDM and MIMO-OFDM (2x2 systems) has been observed. We investigated the LS (Least Square) and MMSE (Minimum mean square error) estimator for both the systems and compare their performances. Based on performance analysis, the MMSE estimator is better in terms of both Mean Square Error (MSE) and Bit Error Rate (BER), but it suffers from high computational complexity. The estimators discussed here can be used to efficiently estimate the channel in SISO-OFDM and MIMO-OFDM systems, given certain knowledge about the channel statistics. The MMSE estimator assumes a priori knowledge of noise variance and channel covariance. We have discussed Space Time Block coding (STBC) and maximum likelihood decoding in the MIMO-OFDM systems to improve its performance by minimizing the probability of error or maximizing the probability of correct detection. Simulation results show that by employing diversity, we can improve the system performance in terms of BER and MSE, simultaneously increasing channel capacity dynamically. Finally, the comparison results reflect that MMSE is more immune to noise for channel estimation but it has complexity issues. On the contrary, LS is simple to implement but suffers from high MSE.

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